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On kaon production in e^+e^- and semi-inclusive DIS reactions

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Abstract. We consider semi-inclusive unpolarized DIS for the production of charged kaons and the different possibilities to test the conventionally used assumptions $s-\bar{s}=0$ and $D_d^{K^+-K^-}=0$. The tests considered have the advantage that they do not require any knowledge of the fragmentation functions. We also show that measurements of both charged and neutral kaons would allow for the determination of the kaon fragmentation functions $D_q^{K^++K^-}$ solely from SIDIS measurements, and we discuss the comparison of $(D_u-D_d)^{K^+-K^-}$ obtained independently in SIDIS and e^+e^- reactions. All analyses are performed in LO and NLO in QCD. The feasibility of the tests to HERMES SIDIS data is considered.

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1 Introduction

It is well known that neutral current inclusive deep inelastic scattering (DIS) yields information only about quark plus antiquark parton densities. When neutrino experiments are possible one can obtain separate knowledge about the quark and antiquark densities, but for the case of polarized DIS this is impossible experimentally. For this case semi-inclusive DIS (SIDIS), where some final hadron is detected, plays an essential role, but it requires knowledge of the fragmentation function (FF) for a given parton to fragment into the relevant hadron. As pointed out in [1] and more recently in [2], precise knowledge of the FFs is vital. In this paper we examine what we can learn about the kaon FFs from the experimental data.

When the spin state of the detected hadron is not monitored, it is possible to learn about the FFs from both $e^+e^- \to hX$ and *unpolarized* SIDIS $l+N \to lhX$. In the case of pion production, SU(2) plays a very helpful role in reducing the number of independent FFs needed. For production of charged kaons, which is important for studying the strange quark densities, SU(2) is less helpful, and even a combined analysis of e^+e^- and SIDIS data on both protons and neutrons does not allow for an unambiguous determination of the kaon FFs [3].

It is thus conventional to make certain reasonable sounding assumptions about the strange quark densities and the kaon FFs. In the first part of this paper we discuss to what extent these assumptions can be justified and tested experimentally. We shall discuss tests based on both a leading order (LO) and a next-to-leading order (NLO) approach. Although it has often been assumed that an NLO treatment is essential, in our paper we have kept the LO treatment for two reasons – one always starts with LO and then follows the natural hierarchy LO → NLO and also because a recent study in [2] showed that a very acceptable description of the combined polarized DIS and SIDIS data can be achieved in a LO approximation as well and thus LO cannot be ruled out yet.

As mentioned above, SU(2) symmetry is of little help if only charged kaons are measured. However, it is well known that charged and neutral kaons are combined in SU(2) doublets. This relates the FFs of K_s^0 to those of $K^\pm$, which implies that no new FFs appear in K_s^0 production. In the second part of our paper we examine to what extent detecting neutral as well as charged kaons can help to determine the kaon fragmentation functions. We carry out the analysis in LO and NLO.

In Sect. 2 we recall the general formulae for inclusive e^+e^- and SIDIS. In Sect. 3 we consider semi-inclusive $K^\pm$ production and possible tests of whether, for the quark densities, $s(x)=\bar{s}(x)$, and of whether, for the fragmentation functions, $D_d^{K^+}(z)=D_d^{K^-}(z)$. In Sect. 4 we discuss production of $K^\pm$, K_s^0; in Sects. 5 and 6 we consider the combinations $K^++K^--2K_s^0$ and $K^++K^-+2K_s^0$ respectively, both in LO and NLO. Possible tests for the

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reliability of the leading order treatment of the processes are discussed.

2 General formulae for e^+e^- and unpolarized SIDIS

For convenience we shall recall some general formulae for the cross sections and asymmetries in $e^+e^- \to hX$ and $e+N \to e+h+X$.

2.1 $e^+e^- \to hX$

There are two distinct measurements of interest: the total cross section $\mathrm{d}\sigma_{\mathrm{T}}^h(z)$ and the forward–backward (FB) asymmetry A_{FB}^h. If $\mathrm{d}^2\sigma^h/(\mathrm{d}z\,\mathrm{d}\cos\theta)$ is the differential cross section for $e^+e^- \to hX$, these quantities are defined by

$$\mathrm{d}\sigma_{\mathrm{T}}^h(z) = \int_{-1}^{+1} \left(\frac{\mathrm{d}^2\sigma^h}{\mathrm{d}z\,\mathrm{d}\cos\theta}\right) \mathrm{d}\cos\theta \tag{1}$$

$$A_{\mathrm{FB}}^h(z) = \left[\int_{-1}^{0} - \int_{0}^{+1}\right] \left(\frac{\mathrm{d}^2\sigma^h}{\mathrm{d}z\,\mathrm{d}\cos\theta}\right) \mathrm{d}\cos\theta\,, \tag{2}$$

where θ is the CM scattering angle, and z is, neglecting masses, the fraction of the momentum of the fragmenting parton transferred to the hadron h: $z = 2(P^h.q)/q^2 = E^h/E$, where E^h and E are the CM energies of the final hadron h and the initial lepton.

From CP invariance it follows that

$$\mathrm{d}\sigma_{\mathrm{T}}^h(z) = \mathrm{d}\sigma_{\mathrm{T}}^{\bar{h}}(z)\,, \quad A_{\mathrm{FB}}^h(z) = -A_{\mathrm{FB}}^{\bar{h}}(z)\,, \tag{3}$$

where $\bar{h}$ is the C-conjugate of the hadron h. Equation (3) implies that the total cross section $\mathrm{d}\sigma_{\mathrm{T}}^h$ actually provides information only about $D_q^{h+\bar{h}} \equiv D_q^h + D_q^{\bar{h}}$, while measurement of A_{FB}^h determines the non-singlet (NS) combinations $D_q^{h-\bar{h}} \equiv D_q^h - D_q^{\bar{h}}$, and this is true in all orders of QCD.

In LO the formulae are especially simple:

$$\mathrm{d}\sigma_{\mathrm{T}}^h(z) = 3\sigma_0 \sum_q \hat{e}_q^2 D_q^{h+\bar{h}}\,, \quad \sigma_0 = \frac{4\pi\alpha_{\mathrm{em}}^2}{3s}\,, \tag{4}$$

$$A_{\mathrm{FB}}^h(z) = 3\sigma_0 \sum_q \frac{3}{2}\hat{a}_q D_q^{h-\bar{h}}\,. \tag{5}$$

Assuming both photon and Z^0-boson exchange, we have

$$\begin{aligned}\hat{e}_q^2(s) &= e_q^2 - 2e_q v_e v_q \,\mathrm{Re}\, h_Z \\ &\quad + \left(v_e^2 + a_e^2\right)\left[(v_q)^2 + (a_q)^2\right]|h_Z|^2 \\ \hat{a}_q &= 2a_e a_q \left(-e_q \,\mathrm{Re}\, h_Z + 2v_e v_q |h_Z|^2\right)\,,\end{aligned} \tag{6}$$

where $h_Z = \left[s/\left(s - m_Z^2 + \mathrm{i} m_Z \Gamma_Z\right)\right]/\sin^2 2\theta_{\mathrm{W}}$. In (6) e_q is the charge of the quark q in units of the proton charge, and, as usual,

$$\begin{aligned} &v_e = -1/2 + 2\sin^2\theta_{\mathrm{W}}\,, \quad a_e = -1/2\,, \\ &v_q = I_3^q - 2e_q \sin^2\theta_{\mathrm{W}}\,, \quad a_q = I_3^q\,, \\ &I_3^u = 1/2\,, \quad I_3^d = -1/2\,. \end{aligned} \tag{7}$$

2.2 Unpolarized SIDIS $e + N \to e + h + X$

In semi-inclusive deep inelastic scattering, we consider the non-singlet difference of the cross sections $\sigma_N^{h-\bar{h}}$, where the measurable quantity is the ratio

$$R_{\mathrm{N}}^{h-\bar{h}} = \frac{\sigma_{\mathrm{N}}^{h-\bar{h}}}{\sigma_{\mathrm{N}}^{\mathrm{DIS}}}\,, \quad \sigma_{\mathrm{N}}^{h-\bar{h}} = \sigma_{\mathrm{N}}^h - \sigma_{\mathrm{N}}^{\bar{h}}\,. \tag{8}$$

For simplicity, we use $\tilde{\sigma}_{\mathrm{N}}^h$ and $\tilde{\sigma}_{\mathrm{N}}^{\mathrm{DIS}}$ in which common kinematic factors have been removed:

$$\tilde{\sigma}_{\mathrm{N}}^h \equiv \frac{x(P+l)^2}{4\pi\alpha^2}\left(\frac{2y^2}{1+(1-y)^2}\right)\frac{\mathrm{d}^3\sigma_{\mathrm{N}}^h}{\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z} \tag{9}$$

$$\tilde{\sigma}_{\mathrm{N}}^{\mathrm{DIS}} \equiv \frac{x(P+l)^2}{4\pi\alpha^2}\left(\frac{2y^2}{1+(1-y)^2}\right)\frac{\mathrm{d}^2\sigma_{\mathrm{N}}^{\mathrm{DIS}}}{\mathrm{d}x\,\mathrm{d}y}\,. \tag{10}$$

Here P and l are the nucleon and lepton four momenta, and x, y, z are the usual deep inelastic kinematic variables: $x = Q^2/2P\cdot q = Q^2/2M\nu$, $y = P\cdot q/P\cdot l = \nu/E$, $z = P\cdot P_h/P\cdot q = E^h/\nu$, where E and E^h are the laboratory energies of the incoming lepton and final hadron. Note that, both in e^+e^- and in SIDIS, neglecting masses, z always measures the fraction of the parton momentum transferred to the produced hadron.

Since the kinematic factors for σ_{N}^h and $\sigma_{\mathrm{N}}^{\mathrm{DIS}}$ are the same, we can write

$$\tilde{\sigma}_{\mathrm{N}}^{h-\bar{h}} = R_{\mathrm{N}}^{h-\bar{h}}\tilde{\sigma}_{\mathrm{N}}^{\mathrm{DIS}}\,, \tag{11}$$

where for $\tilde{\sigma}_{\mathrm{N}}^{\mathrm{DIS}}$ any of the parametrizations for the structure functions F_2 and R or, equivalently, any set of the unpolarized parton densities (PD) can be used.

As shown in [3], the general expression for the cross section differences, in NLO, is given by

$$\begin{aligned}\tilde{\sigma}_p^{h-\bar{h}}(x,z) &= \frac{1}{9}\left[4u_V \otimes D_u^{h-\bar{h}} + d_V \otimes D_d^{h-\bar{h}}\right. \\ &\quad \left. + (s-\bar{s}) \otimes D_s^{h-\bar{h}}\right] \otimes \hat{\sigma}_{qq}(\gamma q \to qX)\,, \\ \tilde{\sigma}_n^{h-\bar{h}}(x,z) &= \frac{1}{9}\left[4d_V \otimes D_u^{h-\bar{h}} + u_V \otimes D_d^{h-\bar{h}}\right. \\ &\quad \left. + (s-\bar{s}) \otimes D_s^{h-\bar{h}}\right] \otimes \hat{\sigma}_{qq}(\gamma q \to qX)\,.\end{aligned} \tag{12}$$

Here $\hat{\sigma}_{qq}$ is the perturbatively calculable, hard partonic cross section $q\gamma^* \to q + X$:

$$\hat{\sigma}_{qq} = \hat{\sigma}_{qq}^{(0)} + \frac{\alpha_{\mathrm{s}}}{2\pi}\hat{\sigma}_{qq}^{(1)}\,, \tag{13}$$

normalized so that $\hat{\sigma}_{qq}^{(0)} = 1$.

It is seen that $\tilde{\sigma}_{\mathrm{N}}^{h-\bar{h}}$ involves only NS parton densities and fragmentation functions, implying that its Q^2 evolution is relatively simple. Equation (12) is sensitive to the valence quark densities, but also to the completely unknown combination $(s-\bar{s})$. The term $(s-\bar{s})D_s^{h-\bar{h}}$ plays no role in pion production, since, by SU(2) invariance, $D_s^{\pi^+-\pi^-} = 0$. However, it is important for kaon production, for which $D_s^{K^+-K^-}$ is a favored transition and thus expected to be big.

Up to now all analyses of experimental data have assumed $s=\bar{s}$. In the next sections we shall consider the production of kaons, $h=K^\pm$ and $h=K^\pm, K_s^0$, and show how this assumption and the assumption $D_d^{K^+-K^-}=0$ can be tested without requiring knowledge of the FFs.

3 Production of charged kaons

As seen from (12), in $R_{\rm N}^{K^+-K^-}$ both $s-\bar{s}$ and $D_d^{K^+-K^-}$ appear. They are expected to be small, and the usual assumption is that they are equal to zero. Here we examine to what extent one can test these assumptions experimentally in SIDIS.

It was shown in [3] that, even if we combine data on the forward–backward asymmetry $A_{\rm FB}^{K^+-K^-}$ in e^+e^- annihilation with measurements of K^+ and K^- production in SIDIS, we cannot determine the fragmentation functions without assumptions. The reason is that we have three measurements for the four unknown quantities $D_{u,d,s}^{K^+-K^-}$ and $(s-\bar{s})$. Thus, one needs an assumption: either $s-\bar{s}=0$ or $D_d^{K^+-K^-}=0$. In fact, up to now, all analyses of experimental data have been performed assuming both $s-\bar{s}=0$ and $D_d^{K^+-K^-}=0$.

Note that from the quark content of $K^\pm$, the assumption $D_d^{K^+-K^-}=0$ seems very reasonable if the $K^\pm$ are directly produced. However, if they are partly produced via resonance decay this argument is less persuasive. Of course, $e^+e^-\to K^\pm X$ sheds no light on this issue.

3.1 LO approximation, $K^\pm$

In LO we have

$$\tilde{\sigma}_p^{K^+-K^-} = \frac{1}{9}\left[4u_V D_u^{K^+-K^-} + d_V D_d^{K^+-K^-} + (s-\bar{s})D_s^{K^+-K^-}\right], \tag{14}$$

$$\tilde{\sigma}_n^{K^+-K^-} = \frac{1}{9}\left[4d_V D_u^{K^+-K^-} + u_V D_d^{K^+-K^-} + (s-\bar{s})D_s^{K^+-K^-}\right]. \tag{15}$$

From a theoretical point of view it is more useful to consider the following combinations of cross sections, which, despite involving differences of cross sections, are likely to be large:

$$(\tilde{\sigma}_p-\tilde{\sigma}_n)^{K^+-K^-} = \frac{1}{9}\left[(u_V-d_V)(4D_u-D_d)^{K^+-K^-}\right],$$
$$(\tilde{\sigma}_p+\tilde{\sigma}_n)^{K^+-K^-} = \frac{1}{9}\left[(u_V+d_V)(4D_u+D_d)^{K^+-K^-} + 2(s-\bar{s})D_s^{K^+-K^-}\right]. \tag{16}$$

We define

$$R_+(x,z) \equiv \frac{(\tilde{\sigma}_p+\tilde{\sigma}_n)^{K^+-K^-}}{u_V+d_V},$$
$$R_-(x,z) \equiv \frac{(\tilde{\sigma}_p-\tilde{\sigma}_n)^{K^+-K^-}}{u_V-d_V}. \tag{17}$$

From a study of the x and z dependence of these we can deduce the following.

1) If $R_-(x,z)$ is a function of z only, then this suggests that an LO approximation is reasonable.
2) If $R_+(x,z)$ is *also* a function of z only, then, since $D_s^{K^+-K^-}$ is a favored transition, we can conclude that $(s-\bar{s})=0$.
3) If $R_+(x,z)$ and $R_-(x,z)$ are *both* functions of z only, and if in addition, $R_+(x,z)=R_-(x,z)$, then both $s-\bar{s}=0$ *and* $D_d^{K^+-K^-}=0$.
4) If $R_+(x,z)$ and $R_-(x,z)$ are *both* functions of z only, but they are *not* equal, $R_+(x,z)\neq R_-(x,z)$, we conclude that $s-\bar{s}=0$, *but* $D_d^{K^+-K^-}\neq 0$.
5) If $R_-(x,z)$ is not a function of z only, then NLO corrections are needed, which we consider below.

The above tests for $s-\bar{s}=0$ and $D_d^{K^+-K^-}=0$ can be spoilt either by $s-\bar{s}\neq 0$ and/or $D_d^{K^+-K^-}\neq 0$, or by NLO corrections, which are both complementary in size. That is why it is important to formulate tests sensitive to $s-\bar{s}=0$ and/or $D_d^{K^+-K^-}=0$ solely, i.e. to consider NLO.

3.2 NLO approximation, $K^\pm$

In an NLO treatment it is still possible to reach some conclusions, though less detailed than in the LO case. We now have

$$(\tilde{\sigma}_p-\tilde{\sigma}_n)^{K^+-K^-} = \frac{1}{9}(u_V-d_V)\otimes(1+\alpha_{\rm s}\mathcal{C}_{qq})\otimes(4D_u-D_d)^{K^+-K^-} \tag{18}$$

$$(\tilde{\sigma}_p+\tilde{\sigma}_n)^{K^+-K^-} = \frac{1}{9}\left[(u_V+d_V)\otimes(4D_u+D_d)^{K^+-K^-} + 2(s-\bar{s})\otimes D_s^{K^+-K^-}\right]\otimes(1+\alpha_{\rm s}\mathcal{C}_{qq}). \tag{19}$$

Here $\mathcal{C}_{ij}$ are

$$\mathcal{C}_{ij}(y) = C_{ij}^M + [1+4\gamma(y)]C_{ij}^{\rm L},$$
$$\gamma(y) = \frac{1-y}{1+(1-y)^2}, \tag{20}$$

$C_{ij}^{M,{\rm L}}$ being the corresponding Wilson coefficients [4]. Suppose we try to fit both (18) and (19) with one and the same fragmentation function $D(z)$,

$$(\tilde{\sigma}_p-\tilde{\sigma}_n)^{K^+-K^-} \approx \frac{4}{9}(u_V-d_V)\otimes(1+\alpha_{\rm s}\mathcal{C}_{qq})\otimes D(z), \tag{21}$$

$$(\tilde{\sigma}_p+\tilde{\sigma}_n)^{K^+-K^-} \approx \frac{4}{9}(u_V+d_V)\otimes(1+\alpha_{\rm s}\mathcal{C}_{qq})\otimes D(z). \tag{22}$$

If this gives an acceptable fit for the x and z dependence of both $p-n$ and $p+n$ data, we can conclude that both $s-\bar{s}\approx 0$ *and* $D_d^{K^+-K^-}\approx 0$ and that $D(z)=D_u^{K^+-K^-}$.

Note that for all above tests, both in LO and NLO approximation, we do not require knowledge of $D_{u,d}^{K^+-K^-}$.

This is especially important since the e^+e^- total cross section data determine only the $D_q^{K^++K^-}$, and these are relatively well known, while $D_{u,d}^{K^+-K^-}$ can be determined solely from $A_{\rm FB}$ in e^+e^- or from SIDIS.

The results of the above tests would indicate what assumptions are reliable in trying to extract the fragmentation functions $D_{u,d,s}^{K^\pm}$ from the same data.

4 Production of charged and neutral kaons

The description of SIDIS and e^+e^- reactions, in which one monitors neutral $K_{\rm s}^0 = (K^0+\bar{K}^0)/\sqrt{2}$ as well as charged $K^\pm$ does not require any further FFs. This is due to SU(2) invariance, which relates the neutral to the charged kaon FFs:

$$\begin{aligned} D_u^{K^++K^-} &= D_d^{K^0+\bar{K}^0}, \quad D_d^{K^++K^-} = D_u^{K^0+\bar{K}^0},\\ D_s^{K^++K^-} &= D_s^{K^0+\bar{K}^0}, \quad D_g^{K^++K^-} = D_g^{K^0+\bar{K}^0}. \end{aligned} \tag{23}$$

In principle, this helps to determine the kaon FFs $D_{u,d,s}^{K^++K^-}$ solely from SIDIS measurements, without the problem of combining e^+e^- data and SIDIS data at widely different values of Q^2.

Two possible measurements can be performed: with $\left(K^++K^--2K_{\rm s}^0\right)$ and with $\left(K^++K^-+2K_{\rm s}^0\right)$.

5 The combination $K^++K^--2K_{\rm s}^0$

In NLO we have

for e^+e^-

$$\begin{aligned} \mathrm{d}\sigma_{\rm T}^{K^++K^--2K_{\rm s}^0}(z) &= 3\sigma_0(\hat{e}_u^2-\hat{e}_d^2)_{m_Z^2}\\ &\quad\times\left[1+\frac{\alpha_{\rm s}}{2\pi}C_F(c_{\rm T}^q+c_{\rm L}^q)\otimes\right]\\ &\quad\times(D_u-D_d)^{K^++K^-},\\ \mathrm{d}\sigma_{\rm T}^{K^++K^--2K_{\rm s}^0} &\equiv \mathrm{d}\sigma_{\rm T}^{K^+}+\mathrm{d}\sigma_{\rm T}^{K^-}-2\mathrm{d}\sigma_{\rm T}^{K_{\rm s}^0}, \end{aligned} \tag{24}$$

where $c_{\rm T,L}^q$ are the Wilson coefficients for the contribution of the transverse (T) and longitudinal (L) virtual boson [5].

For SIDIS we have

$$\begin{aligned} &\tilde{\sigma}_p^{K^++K^--2K_{\rm s}^0}(x,y,z)\\ &= \left\{\frac{1}{9}[4(u+\bar{u})-(d+\bar{d})]\left(1+\frac{\alpha_{\rm s}}{2\pi}\otimes\mathcal{C}_{qq}\otimes\right)\right.\\ &\left.\quad+\frac{1}{3}\frac{\alpha_{\rm s}}{2\pi}g\otimes\mathcal{C}_{gq}\otimes\right\}(D_u-D_d)^{K^++K^-}, \end{aligned} \tag{25}$$

$$\begin{aligned} &\tilde{\sigma}_n^{K^++K^--2K_{\rm s}^0}(x,y,z)\\ &= \left\{\frac{1}{9}[4(d+\bar{d})-(u+\bar{u})]\left(1+\frac{\alpha_{\rm s}}{2\pi}\otimes\mathcal{C}_{qq}\otimes\right)\right.\\ &\left.\quad+\frac{1}{3}\frac{\alpha_{\rm s}}{2\pi}g\otimes\mathcal{C}_{gq}\otimes\right\}(D_u-D_d)^{K^++K^-}. \end{aligned} \tag{26}$$

Thus, due to SU(2) invariance, in all orders of QCD all three processes always measure the same NS combination of fragmentation functions $(D_u-D_d)^{K^++K^-}$, whose evolution does not involve the very poorly known gluon fragmentation functions.

The difference of cross sections $K^++K^--2K_{\rm s}^0$, involving neutral kaons, is essential in order to eliminate, due to SU(2) invariance, the $s+\bar{s}$-quark parton densities and the gluon FF.

Note that the combinations of quark densities in the above *do* have a singlet component and thus depend on $g(x)$, but that is not a problem.

5.1 LO approximation, $K^++K^--2K_{\rm s}^0$

The LO expressions are particularly simple; they are obtained from (24)–(26) with $\alpha_{\rm s}=0$. They imply that SIDIS determines $(D_u-D_d)^{K^++K^-}$ given $(u+\bar{u})$ and $(d+\bar{d})$ are known.

The difference $\tilde{\sigma}_p-\tilde{\sigma}_n$ is

$$\begin{aligned} &(\tilde{\sigma}_p-\tilde{\sigma}_n)^{K^++K^--2K_{\rm s}^0}(x,y,z)\\ &\quad=\frac{5}{9}[(u+\bar{u})-(d+\bar{d})](D_u-D_d)^{K^++K^-}, \end{aligned} \tag{27}$$

which is a non-singlet in both the PDs and the FFs. This implies that in its Q^2 evolution and in all orders in QCD it will always contain the same NS combinations, convoluted with the corresponding Wilson coefficients when higher orders are considered.

The fact that e^+e^- and SIDIS measure the same combination $(D_u-D_d)^{K^++K^-}$ allows one to combine e^+e^- data at $Q^2\simeq m_Z^2$, where Z^0-exchange is the dominant contribution, with SIDIS experiments at $Q^2\ll m_Z^2$, where γ-exchange dominates. For example, one could test the relation

$$\frac{9\mathrm{d}\tilde{\sigma}_p^{K^++K^--2K_{\rm s}^0}(x,z,Q^2)}{\mathrm{d}\sigma_{\rm T}^{K^++K^--2K_{\rm s}^0}(z,m_Z^2)_{\downarrow Q^2}} = \frac{[4(u+\bar{u})-(d+\bar{d})](x,Q^2)}{3\sigma_0\,(\hat{e}_u^2-\hat{e}_d^2)_{m_Z^2}}. \tag{28}$$

Here $\mathrm{d}\sigma_{\rm T}^{K^++K^--2K_{\rm s}^0}(z,m_Z^2)_{\downarrow Q^2}$ denotes that the data are measured at m_Z^2 and then evolved to Q^2 according to the DGLAP equations. This would be a test of LO, but also a test of the factorization of SIDIS into parton densities times FFs.

Tests for whether LO is a reasonable approximation for the SIDIS reactions can be made as follows. In LO one should have

1) for proton targets

$$\begin{aligned} \frac{\tilde{\sigma}_p^{K^++K^--2K_{\rm s}^0}(x,z)}{4(u+\bar{u})-(d+\bar{d})} &= \text{function of } z \text{ only}\\ &\equiv f_p(z) = (D_u-D_d)^{K^++K^-}(z), \end{aligned} \tag{29}$$

2) for neutron targets

$$\frac{\tilde{\sigma}_n^{K^++K^--2K_s^0}(x,z)}{4(d+\bar{d})-(u+\bar{u})} = \text{function of } z \text{ only}$$
$$\equiv f_n(z) = (D_u - D_d)^{K^++K^-}(z)\,, \tag{30}$$

where the PDs are determined in LO (see for example [6]);

3) and if measurements for both proton and neutron targets are available, then also

$$f_p(z) = f_n(z) \tag{31}$$

should hold, as expected from (29) and (30).

The above LO tests do not require knowledge of the FFs. Concerning the measurement of FFs, an attempt was made in [1] to combine data on e^+e^- and SIDIS. The evolution involved there required an estimate of the gluon FF that induced quite large errors. In the present case, we study the NS combination $(D_u - D_d)^{K^++K^-}$, which can be measured both in e^+e^- and SIDIS, (24)–(26), and whose evolution in Q^2 is straightforward, since it does not involve the gluon FFs.

5.2 NLO approximation, $K^+ + K^- - 2K_s^0$

In higher orders of QCD the cross sections on p and n with $K^+ + K^- - 2K_s^0$ depend on the gluon PD – see (25) and (26). The difference of the cross sections on proton and neutron eliminates $g(x)$:

$$(\tilde{\sigma}_p - \tilde{\sigma}_n)^{K^++K^--2K_s^0}(x,y,z)$$
$$= \frac{5}{9}[(u+\bar{u})-(d+\bar{d})]\left(1+\frac{\alpha_s}{2\pi}\otimes\mathcal{C}_{qq}\otimes\right)(D_u - D_d)^{K^++K^-}\,, \tag{32}$$

and (32) determines $(D_u - D_d)^{K^++K^-}$ without the influence of even the gluon quarks or any other FF. Note that $(u+\bar{u})-(d+\bar{d})$ is an NS and thus $g(x)$ will not creep back through the Q^2 evolution.

Further, being a NS it would not be a problem to compare the two independent measurements: in e^+e^- annihilation at $Q^2 \simeq m_Z^2$, (24), and in SIDIS at $Q^2 \ll m_Z^2$, (32). They should give the same result, when evolved to the same Q^2 according to the DGLAP equations, and thus present a test of the hypothesis that SIDIS is a product of the quark production and quark fragmentation processes. This test would be independent of the gluon and strange PDs or any other FFs and hold in any order in QCD.

Having thus determined $(D_u - D_d)^{K^++K^-}$ one may proceed to determine the gluon PD, without the uncertainties of $s+\bar{s}$, measuring the sum of the same cross sections on p and n:

$$(\tilde{\sigma}_p + \tilde{\sigma}_n)^{K^++K^--2K_s^0}(x,y,z)$$
$$= \frac{1}{3}\left\{[(u+\bar{u})+(d+\bar{d})]\left(1+\frac{\alpha_s}{2\pi}\otimes\mathcal{C}_{qq}\otimes\right)+2\frac{\alpha_s}{2\pi}g\otimes\mathcal{C}_{gq}\otimes\right\}$$
$$\times(D_u - D_d)^{K^++K^-}\,. \tag{33}$$

6 The combination $K^+ + K^- + 2K_s^0$

The general expressions in NLO are rather lengthy, so we begin by discussing the LO case, which already exhibits the main properties. For brevity we use the notation $(K) \equiv K^+ + K^- + 2K_s^0$.

6.1 LO approximation, $K^+ + K^- + 2K_s^0$

In LO we have

for e^+e^-

$$\mathrm{d}\sigma_{\mathrm{T}}^{(K)}(z) = 3\sigma_0\left[(\hat{e}_u^2+\hat{e}_d^2)_{m_Z^2}(D_u+D_d)^{K^++K^-}\right.$$
$$\left.+2\hat{e}_d^2 D_s^{K^++K^-}\right]\,, \tag{34}$$
$$\mathrm{d}\sigma_{\mathrm{T}}^{(K)} \equiv \mathrm{d}\sigma_{\mathrm{T}}^{K^+} + \mathrm{d}\sigma_{\mathrm{T}}^{K^-} + 2\mathrm{d}\sigma_{\mathrm{T}}^{K_s^0}\,,$$

and for SIDIS

$$\tilde{\sigma}_p^{(K)}(x,z,Q^2)$$
$$= \frac{1}{9}\left[(4(u+\bar{u})+(d+\bar{d}))(D_u+D_d)^{K^++K^-}\right.$$
$$\left.+2(s+\bar{s})D_s^{K^++K^-}\right] \tag{35}$$
$$\tilde{\sigma}_n^{(K)}(x,z,Q^2)$$
$$= \frac{1}{9}\left[(4(d+\bar{d})+(u+\bar{u}))(D_u+D_d)^{K^++K^-}\right.$$
$$\left.+2(s+\bar{s})D_s^{K^++K^-}\right]\,. \tag{36}$$

Equations (34)–(36) imply that due to SU(2) invariance, the three cross sections $\mathrm{d}\sigma_{\mathrm{T}}^{(K)}$, $\tilde{\sigma}_p^{(K)}$ and $\tilde{\sigma}_n^{(K)}$ always measure only two combinations of FFs: $(D_u + D_d)^{K^++K^-}$ and $D_s^{K^++K^-}$. Note that, as this is a property of the SU(2) symmetry, it will hold in all orders of QCD; only the gluon FF will enter in addition in higher orders.

From (34)–(36) it follows that in LO we have three measurements for two unknown quantities: $(D_u + D_d)^{K^++K^-}$ and $D_s^{K^++K^-}$. This implies in particular that measurements of $K^+ + K^- - 2K_s^0$ and $K^+ + K^- + 2K_s^0$ in SIDIS – see (25), (26), (35) and (36) – are already enough to determine $(D_u \pm D_d)^{K^++K^-}$ and $D_s^{K^++K^-}$, and it is not necessary to use data from e^+e^- performed at very different Q^2.

The difference $\tilde{\sigma}_p^{(K)} - \tilde{\sigma}_n^{(K)}$ determines $(D_u + D_d)^{K^++K^-}$ only through the NS combination $(u+\bar{u})-(d+\bar{d})$:

$$\tilde{\sigma}_p^{(K)} - \tilde{\sigma}_n^{(K)} = \frac{1}{3}[(u+\bar{u})-(d+\bar{d})](D_u+D_d)^{K^++K^-}\,. \tag{37}$$

Once we have thus determined $(D_u + D_d)^{K^++K^-}$, we can use $\tilde{\sigma}_{p,n}^{(K)}$ (or equivalently their sum $\tilde{\sigma}_p^{(K)} + \tilde{\sigma}_n^{(K)}$) to obtain $D_s^{K^++K^-}$.

Only in LO SIDIS measurements are enough to determine $D_{u,d,s}^{K^++K^-}$. It is thus important to have reliable tests of the LO approximation. It is an advantage that using the same expressions (35)–(36) one can form possible tests of the LO in these processes.

1) In LO we have

$$\frac{3(\tilde{\sigma}_p-\tilde{\sigma}_n)^{K^++K^-+2K_{\rm s}^0}(x,z)}{\left(u+\bar{u}-(d+\bar{d})\right)(x)} = \text{ function of } z \text{ only}$$
$$= (D_u+D_d)^{K^++K^-}(z)\,. \qquad (38)$$

2) If the $K_{\rm s}^0$ are not measured, LO would be a good approximation if

$$\frac{9(\tilde{\sigma}_p-\tilde{\sigma}_n)^{K^++K^-}(x,z)}{\left(u+\bar{u}-(d+\bar{d})\right)(x)} = \text{function of } z \text{ only}$$
$$= (4D_u-D_d)^{K^++K^-}(z)\,, \qquad (39)$$

i.e. only the combination of FFs on the r.h.s. is different from (38).

For neither of these tests is knowledge of the FFs necessary, and they should lead to the extraction of $D_q^{K^++K^-}$.

6.2 NLO approximation, $K^+ + K^- + 2K_{\rm s}^0$

As mentioned, in NLO the three cross sections $\mathrm{d}\sigma_{\rm T}^{(K)}$, $\tilde{\sigma}_p^{(K)}$ and $\tilde{\sigma}_n^{(K)}$ measure different combinations of the three unknown FFs:

$$(D_u+D_d)^{K^++K^-}, \quad D_s^{K^++K^-}, \quad D_g^{K^++K^-}. \qquad (40)$$

(The general expressions for NLO are rather lengthy, so we present below only those relevant for our discussion.) This implies that in NLO, contrary to LO, both e^+e^- and SIDIS measurements are needed to determine (40). Combined with measurements of $K^++K^--2K_{\rm s}^0$, (24)–(26), we have enough measurements to determine all kaon FFs: $(D_u\pm D_d)^{K^++K^-}$, $D_s^{K^++K^-}$ and $D_g^{K^++K^-}$.

Solely from SIDIS, and without the influence of the strange and gluon PDs, in NLO one can determine $D_{u\pm d}^{K^++K^-}$ and $D_g^{K^++K^-}$. The difference of $(\tilde{\sigma}_p-\tilde{\sigma}_n)^{(K)}$ determines a combination of $(D_u+D_d)^{K^++K^-}$ and $D_g^{K^++K^-}$, where the PDs enter only as a common factor in the combination $(u+\bar{u})-(d+\bar{d})$:

$$\begin{aligned}&(\tilde{\sigma}_p-\tilde{\sigma}_n)^{K^++K^-+2K_{\rm s}^0}(x,y,z)\\&=\frac{1}{3}[(u+\bar{u})-(d+\bar{d})]\left\{\left[1+\frac{\alpha_{\rm s}}{2\pi}\otimes\mathcal{C}_{qq}\otimes\right]\right.\\&\quad\left.\times(D_u+D_d)^{K^++K^-}+2\frac{\alpha_{\rm s}}{2\pi}\otimes\mathcal{C}_{qg}\otimes D_g^{K^++K^-}\right\}.\end{aligned} \qquad (41)$$

As these FFs are not NS and thus have a different Q^2 evolution, the above equation would provide information on both $(D_u+D_d)^{K^++K^-}$ and $D_g^{K^++K^-}$. Further, combined with measurements on $(D_u-D_d)^{K^++K^-}$ from (32), one can determine $(D_u\pm D_d)^{K^++K^-}$ and $D_g^{K^++K^-}$ in NLO solely in SIDIS, and they will depend on the parton densities only through the combination $(u+\bar{u})-(d+\bar{d})$.

Further one can combine the measurements of $D_{u\pm d}^{K^++K^-}$ and $D_g^{K^++K^-}$ with measurements of e^+e^- annihilation or the $p+n$ SIDIS cross section to determine $D_s^{K^++K^-}$. Especially useful would be e^+e^- annihilation where $D_s^{K^++K^-}$ is not multiplied by the small quantity $(s+\bar{s})$:

$$\begin{aligned}&\mathrm{d}\sigma_{\rm T}^{(K)}(z)\\&=3\sigma_0\left\{\left(\left(\hat{e}_u^2+\hat{e}_d^2\right)_{m_Z^2}(D_u+D_d)^{K^++K^-}+2\hat{e}_d^2D_s^{K^++K^-}\right)\right.\\&\quad+\left[1+\frac{\alpha_{\rm s}}{2\pi}\otimes C_F\left(c_{\rm T}^q+c_{\rm L}^q\right)\right]\\&\quad\left.+2\frac{\alpha_{\rm s}}{2\pi}\left(\hat{e}_u^2+2\hat{e}_d^2\right)_{m_Z^2}\otimes C_F\left(c_{\rm T}^g+c_{\rm L}^g\right)D_g^{K^++K^-}\right\}.\end{aligned}$$

The advantage is that in this way neither the strange nor the gluon parton densities influence the determination of the kaon FFs.

In summary, if in addition to the charged $K^\pm$ also the neutral $K_{\rm s}^0$ are measured, we showed that in LO all FFs $D_{u,d,s}^{K^++K^-}$ can be determined solely from SIDIS, i.e. it is not necessary to use data from e^+e^- performed at very different Q^2. In NLO e^+e^- data should be included, as well, and then all FFs can be determined without the influence of the strange and gluon PDs. The non-singlet $(D_u-D_d)^{K^++K^-}$ can be singled out in both e^+e^- and SIDIS. Since comparing the two measurements at different Q^2 is straightforward, one can test the factorization of the SIDIS cross section into parton densities and fragmentation functions both in LO and NLO.

7 Conclusions

The paper considers the possibilities to obtain the kaon FFs in e^+e^- annihilation and SIDIS. It consists of two parts. In the first part we have considered possible tests for $s-\bar{s}=0$ and $D_d^{K^+-K^-}=0$ in unpolarized SIDIS with final charged $K^\pm$, both in LO and NLO of QCD.

In the second part we have shown that, if in addition to $K^\pm$ also the neutral $K_{\rm s}^0$ are measured 1) in LO the kaon FFs can be obtained solely from SIDIS, and 2) in NLO the combined data of the total cross section in e^+e^- annihilation in addition to SIDIS is also needed; then the FFs can be determined without the uncertainties of the strange and gluon PDs. Different possibilities to test the LO approximation in unpolarized SIDIS are discussed and in all proposed tests no knowledge of the fragmentation functions is necessary. We show that, in all orders of QCD, the non-singlet combination $(D_u-D_d)^{K^++K^-}$ can be measured directly both in e^+e^- and in SIDIS without any influence of the strange and gluon parton densities or any other FFs. Comparing the measurements in e^+e^- and SIDIS allows for tests of the factorization of SIDIS into parton densities and fragmentation functions in any order in QCD.

In our approach we consider the sum and difference of cross sections for hadron h and its C-conjugate $\bar{h}$. The cross section differences, $h-\bar{h}$, are NS, and both their Q^2 evolution and NLO corrections in QCD are straightforward, since they do not mix with other PDs or FFs. But they involve poorly known quantities such as the non-singlets $s-\bar{s}$ and $D_d^{K^+-K^-}$, and we suggest some tests for these quantities. Quite the opposite is true when the sum of cross sections $h+\bar{h}$ is considered. In this case the Q^2 evolution and NLO corrections involve the poorly known gluon FFs, but the cross sections contain the best known combinations of PDs $q+\bar{q}$, measured in DIS, and $D_q^{h^++h^-}$ measured in e^+e^-.

We have tried to exploit some of the advantages of both types of combinations of data. Note that, though we often consider difference asymmetries, the quantities that they determine are not small and thus, we hope, measurable.

We want to add a few remarks on the measurability of the discussed asymmetries. In general, these are difference asymmetries and high precision measurements are required. In addition, the data should be presented in bins in both x and z. Quite recently such binning was done in [7] for the very precise data of the HERMES collaboration in DESY on $K^\pm$ production in semi-inclusive DIS on proton and deuterium targets. These results show that for $0.350 \le z \le 0.450$ and for $0.450 \le z \le 0.600$ in the x-interval $0.023 \le x \le 0.300$ the accuracy of the data allows us to form the differences $(\sigma_p+\sigma_n)^{K^+-K^-}$ and $(\sigma_p-\sigma_n)^{K^+-K^-}$ with errors not bigger than 7%–13% and 10%–15% respectively. Having these cross sections, given that u_V and d_V are well known, one can form the ratios R_+ and R_- with these precisions. Then, if we do not obtain an acceptable fit to $R_+(x,z_0)$ that is independent of x, then $s-\bar{s}=0$ is not a good approximation. This conclusion assumes the success of the LO test involving $R_-(x,z_0)$ and is independent of our knowledge of the FFs.

If, however, an acceptable x independent fit to $R_+(x,z_0)$ is obtained, then the precision of this fit will put limits on $(s-\bar{s})D_s^{K^+-K^-}$. Using these limits in the expression for R_+-R_- and comparing it with experiment at the same values z_0 will then put limits on $D_d^{K^+-K^-}$.

If we work in NLO and do not succeed in obtaining an acceptable fit for (21) and (22) with the same $D(z)$, then $s-\bar{s}\simeq 0$ and $D_d^{K^+-K^-}\simeq 0$ cannot hold simultaneously; at least one of these assumptions fails.

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